Off-equilibrium response function in the one-dimensional random-field Ising model

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A thorough numerical investigation of the slow dynamics in the $d=1$ random-field Ising model in the limit of an infinite ferromagnetic coupling is presented in this paper. Crossovers from the preasymptotic pure regime to the asymptotic Sinai regime are investigated for the average domain size, the autocorrelation function, and staggered magnetization. By switching on an additional small random field at the time t_w the linear off-equilibrium response function is obtained, which displays as well the crossover from the nontrivial behavior of the $d=1$ pure Ising model to the asymptotic behavior where it vanishes identically.

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I. INTRODUCTION

A phase-ordering system, such as a ferromagnet quenched below the critical point, offers the simplest example of slow relaxation with many of the interesting features observed also in more complex glassy systems [1]. At the core of this phenomenology is the separation of the time scales of the fast and slow variables. In a domain-forming system equilibrium is rapidly reached in the interior of domains while the interfacial degrees of freedom remain out of equilibrium for a time that diverges with the size of the system. In these conditions the order-parameter autocorrelation function splits into the sum

$$G(t,t_w)=G_{\mathrm{st}}(t-t_w)+G_{\mathrm{ag}}(t/t_w) \tag{1}$$

where $G_{\mathrm{st}}(t-t_w)$ is the stationary time translation invariant contribution due to fluctuations in the bulk of domains and $G_{\mathrm{ag}}(t/t_w)$ is the aging, or scaling contribution [2,3] originating in the off-equilibrium fluctuations. In the time scale over which $G_{\mathrm{st}}(t-t_w)$ decays to zero $G_{\mathrm{ag}}(t/t_w)$ stays practically constant at M^2, where M is the equilibrium value of the order parameter, while the decay of $G_{\mathrm{ag}}(t/t_w)$ takes place for much larger time separations. Using $G(t,t)=1$ for spin variables, this implies $G_{\mathrm{st}}(0)=1-M^2$.

A structure similar to Eq. (1) is observed also in the linear response at the time t to a random external field switched on at the earlier time t_w

$$\chi(t,t_w)=\chi_{\mathrm{st}}(t-t_w)+\chi_{\mathrm{ag}}(t,t_w), \tag{2}$$

where the stationary contributions $\chi_{\mathrm{st}}(t-t_w)$ and $G_{\mathrm{st}}(t-t_w)$ are related by the equilibrium fluctuation dissipation theorem

$$T\chi_{\mathrm{st}}(t-t_w)=G_{\mathrm{st}}(0)-G_{\mathrm{st}}(t-t_w) \tag{3}$$

and $\chi_{\mathrm{ag}}(t,t_w)$ is the off-equilibrium extra response. Notice that Eq. (3) may be read as the statement that $\chi_{\mathrm{st}}(t-t_w)$ depends on time through $G_{\mathrm{st}}(t-t_w)$. Mean-field theory for glassy systems predicts [4] that in the asymptotic time region this holds also off-equilibrium with $\chi(t,t_w)=\hat{\chi}(G(t,t_w))$. If this is the case and if $\lim_{t\to\infty}\chi(t,t_w)=\chi_{\mathrm{eq}}$ then static and dynamic properties are connected [5] by the relation

$$-T\left.\frac{d^2\hat{\chi}(G)}{dG^2}\right|_{G=q}=P_{\mathrm{eq}}(q), \tag{4}$$

where χ_{eq} and $P_{\mathrm{eq}}(q)$ are the linear susceptibility and the overlap probability distribution [6] in the equilibrium state. Applying the scheme to phase ordering, we should find

$$-T\left.\frac{d^2\hat{\chi}(G)}{dG^2}\right|_{G=q}=\delta(q-M^2) \tag{5}$$

since the overlap function in the equilibrium state is given by the δ function [6]. On the other hand Eq. (3) may be rewritten as $T\chi_{\mathrm{st}}(t-t_w)=(1-[G_{\mathrm{st}}(t-t_w)+M^2])$ and, if t_w is sufficiently large, parametrizing time through the full autocorrelation function we have

$$T\hat{\chi}_{\mathrm{st}}(G)=\begin{cases}1-G & \text{for } M^2\leqslant G\leqslant 1\\ 1-M^2 & \text{for } G<M^2,\end{cases} \tag{6}$$

which gives $-T(d^2/dG^2)\hat{\chi}_{\mathrm{st}}(G)|_{G=q}=\delta(q-M^2)$. Hence, for Eq. (4) to hold, one must necessarily have

$$\lim_{t_w\to\infty}\chi(t,t_w)=\hat{\chi}_{\mathrm{st}}(G) \tag{7}$$

and $\lim_{t_w\to\infty}\chi_{\mathrm{ag}}(t,t_w)=0$. Numerical simulations for the Ising model [7] in $d=2$ and $d=3$ as well as analytical results for the spherical model [8] indeed show evidence that Eq. (7) is asymptotically satisfied. More precisely, one expects [9,10] the off-equilibrium contribution to the response

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function to scale as $\chi_{ag}(t,t_w)=t_w^{-a}\tilde{\chi}_{ag}(t/t_w)$ with $a=1/2$ on the basis of the argument that $\chi_{ag}(t,t_w)$ is proportional to the interface density $\rho_I(t)\sim L^{-1}(t)$ where $L(t)\sim t^{1/z}$ is the typical domain size and $z=2$ for nonconserved order-parameter dynamics [3].

Motivated by analytical results [11] for the one-dimensional Ising model, which, instead, give $a=0$, recently a detailed study of the behavior of the response function has been undertaken [12] wherein an interesting dependence of the exponent a on the space dimensionality has been seen. This is best explained in terms of the effective response $\chi_{eff}(t,t_w)$ due to a single interface and defined by

$$\chi_{ag}(t,t_w)=\rho_I(t)\chi_{eff}(t,t_w). \tag{8}$$

In this form the behavior of $\chi_{ag}(t,t_w)$ appears to be determined by the balance between the interface loss due to coarsening and the response associated to a single interface. If one requires $\chi_{ag}(t,t_w)\sim\rho_I(t)$ clearly $\chi_{eff}(t,t_w)$ must be a constant. In Ref. [12] it was found that for an Ising system this is the case only for $d>3$, while for $d<3$ there is the power-law growth

$$\chi_{eff}(t,t_w)\sim(t-t_w)^{\alpha} \tag{9}$$

with numerical values for the exponent at different dimensionalities compatible with $\alpha=(3-d)/4$. At $d=3$ the power law is replaced by logarithmic growth. It has been conjectured that this dimensionality dependence of α is the outcome of the competition between the curvature of interfaces and the external perturbing field in the drive of interface motion. According to this picture $d=3$ is the dimensionality above which interface motion is dominated by the curvature, while below $d=3$ the external field competes effectively with the curvature. The more so, the lower is the dimensionality, until in $d=1$ interfaces reduce to pointlike objects driven only by the external field. When this happens, the rate of growth of the single interface response matches exactly the rate of loss of the interface density, with $\alpha=1/2$ and $a=0$. Then, $\chi_{ag}(t,t_w)$ does not vanish as $t_w\to\infty$ and the second term in Eq. (2) contributes to $\hat{\chi}(G)$ producing the violation of Eq. (4). In short, dimensionality acts as the control parameter that allows to modulate the competition between the two opposing mechanisms driving interface motion and is summarized by

$$a=\begin{cases}\frac{1}{2} & \text{for } d>3\\ \frac{d-1}{4} & \text{for } d\leqslant 3.\end{cases} \tag{10}$$

In this paper we investigate a generalization of this phenomenon occurring in the framework of the random-field Ising model (RFIM), which further clarifies how the overall behavior of $\chi_{ag}(t,t_w)$ originates in the interplay between the rate of growth of the single interface response function and the rate of loss of defect density. Here we specialize to the $d=1$ RFIM. In this case domain walls perform random walks in a random potential of the Sinai type [13]. The crucial feature of this walk, from our perspective, is the crossover between the preasymptotic regime, characterized by free diffusion as in the pure system, and the asymptotic regime characterized by Sinai diffusion. Measuring $\chi_{eff}(t,t_w)$ and $\rho_I(t)$, in the preasymptotic regime we shall find

$$\chi_{eff}(t,t_w)\geqslant\rho_I^{-1}(t) \tag{11}$$

as in the pure system. Namely, as long as diffusion takes place in a flat landscape, the rate of growth of $\chi_{eff}(t,t_w)$ does not go below the rate of loss of interfaces. However, going over to the asymptotic regime the landscape becomes rugged and activated processes do play a role. When this happens $\chi_{eff}(t,t_w)$ slows down with respect to $\rho_I(t)$ eventually reaching

$$\chi_{eff}(t,t_w)\sim\rho_I^{-1/2}(t), \tag{12}$$

which leads to the vanishing of $\chi_{ag}(t,t_w)$. Through this mechanism the validity of Eq. (4) is restored in the $d=1$ RFIM.

II. UNPERTURBED SYSTEM

In the following we consider the $d=1$ Ising model with the Hamiltonian

$$\mathcal{H}[\sigma_i]=-J\sum_{i=1}^{N}\sigma_i\sigma_{i+1}-\sum_{i=1}^{N}h_i\sigma_i, \tag{13}$$

where $J>0$ is the ferromagnetic coupling and $h_i=\pm h_0$ is an uncorrelated random field with expectations

$$E_h(h_i)=0$$

$$E_h(h_ih_j)=h_0^2\delta_{ij}. \tag{14}$$

Here h_i is not a perturbing field, so h_0 does not have to be small. We will consider the two different dynamical evolutions taking place when the system is quenched to the final temperature T starting from the two initial states.

(i) The system is prepared into a spin configuration containing a single interface

$$\sigma_i=\mathrm{sgn}(i) \tag{15}$$

(ii) The system is in equilibrium at infinite temperature with the uniform measure

$$\mathcal{P}_0[\sigma_i]=2^{-N}. \tag{16}$$

In order to have a phase-ordering process the equilibrium state at the final temperature of the quench must be ordered. In the $d=1$ RFIM this is not possible even at $T=0$, since the size of the ordered domains is limited by the Imry Ma length $L_{IM}=4J^2/h_0^2$. So, we take the limit [12] of an infinite ferromagnetic coupling $J\to\infty$ in order to have $L_{IM}=\infty$. In this case the equilibrium state is the mixture of the two ordered states

$$\mathcal{P}_{\rm eq}[\sigma_i]=\frac{1}{Z}\exp\left(-\frac{1}{T}\mathcal{H}[\sigma_i]\right)\times\frac{1}{2}\prod_i\delta(\sigma_i-1)+\frac{1}{2}\prod_i\delta(\sigma_i+1)\qquad(17)$$

for any finite temperature T. Notice that in the corresponding pure states $M^2=1$, which implies $\chi_{\rm eq}=0$ and $P_{\rm eq}(q)=\delta(q-1)$. Furthermore, with $J=\infty$ thermal fluctuations within domains are suppressed and dynamics is reduced to interface diffusion. With the initial condition (15) we have the diffusion of the single interface and with the initial condition (16) we get the diffusion-annihilation process of the set of interfaces seeded by the initial condition. Characteristic lengths of the process are in the first case the root-mean-square displacement (RMSD) $R(t)$ of the single interface and in the second case the average distance between interfaces (or average domain size) $L(t)$. Furthermore, since the typical size of the potential barrier that a walker must overcome after traveling a distance l is $l^{1/2}h_0$, there remains defined another characteristic length

$$L_g=\left(\frac{T}{h_0}\right)^2\qquad(18)$$

as the distance over which potential barriers are of the order of magnitude of thermal energy. This length is important because it separates the distances smaller than L_g, over which diffusion takes place as in the pure system with $R(t)$ or $L(t)\sim t^{1/2}$, from the distances larger than L_g, over which diffusion is dominated by the random potential and it is of the Sinai type [14] with $R(t)$ or $L(t)\sim(\ln t)^2$. Clearly, the limit of the pure system corresponds to $L_g=\infty$.

Considering different temperatures T in the presence of random fields of different strength h_0, we have let the system evolve with initial conditions (15) and (16). One time unit is defined equal to one flip attempt per spin, on the average (spins are updated in random order). We use Metropolis transition rates, which is the probability to flip the spin is $p_{\rm flip}=\min(1,\exp[-(1/T)\Delta\mathcal{H}])$. In the case of initial condition (15), we have found that the RMSD of the single interface depends on T and h_0 through L_g satisfying (Fig. 1) the scaling relation

$$R(t,L_g)=L_g\mathcal{R}\left(\frac{t}{L_g^2}\right)\qquad(19)$$

with

$$\mathcal{R}(x)\sim\begin{cases}x^{1/2} & \text{for } x\ll 1\\(\ln x)^2 & \text{for } x\gg 1,\end{cases}\qquad(20)$$

which displays the crossover from the pure regime to the Sinai regime. A completely analogous behavior is obtained in the second case for the typical domain size $L(t)$, measured as the inverse density of interfaces $\rho_I^{-1}(t)$. We find (Fig. 2)

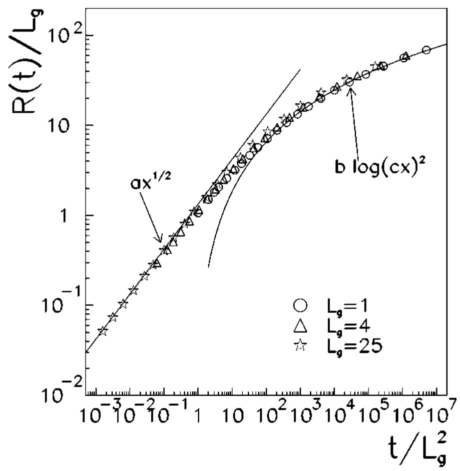

FIG. 1. Rescaled RMSD $R(t)/L_g$ versus rescaled time t/L_g^2, for a single interface and $L_g=1,4,25$. The fit parameters are $a=1.33$, $b=0.274$, and $c=1.33$.

$$L(t,L_g)=L_g\mathcal{L}\left(\frac{t}{L_g^2}\right),\qquad(21)$$

where the scaling function $\mathcal{L}(x)$ obeys the same limiting behaviors as in Eq. (20).

Next, let us consider the autocorrelation function $G(t,t_w,L_g)=(1/N)E_h[\Sigma_i\langle\sigma_i(t)\sigma_i(t_w)\rangle_h]$ where the angular brackets denote the average over thermal noise for a given realization of the field $[h_i]$. Due to the absence of thermal fluctuations within domains $G_{\rm st}(t-t_w)\equiv 0$ and the autocorrelation function is entirely given by the aging component that satisfies (Fig. 3) the scaling relation

$$G_{\rm ag}(t,t_w,L_g)=F\left(\frac{L(t)}{L(t_w)},z\right),\qquad(22)$$

where $z=L(t_w)/L_g$. The shape of the scaling function $F(x,z)$ is known in the limits of the pure system ($z=0$) and of the Sinai regime ($z=\infty$). In the first case from Eq. (21)

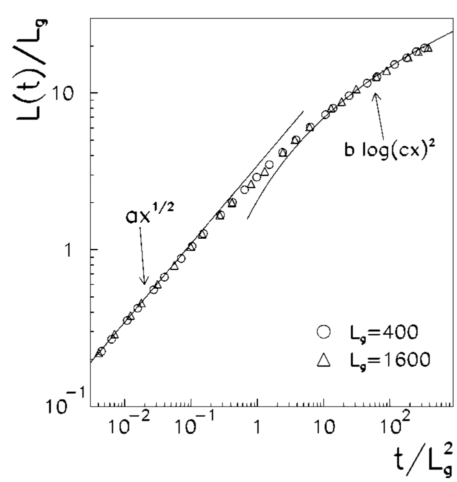

FIG. 2. Rescaled average length of domains $L(t)/L_g$ versus rescaled time t/L_g^2, for multiple interfaces and $L_g=400,1600$. The fit parameters are $a=3.45$, $b=0.271$, and $c=15.8$.

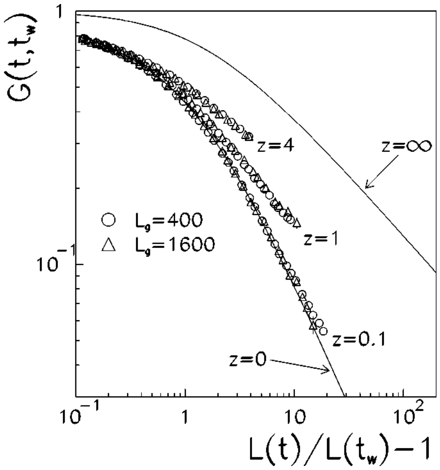

FIG. 3. Autocorrelation function $G(t,t_w)$ versus the ratio $[L(t)-L(t_w)]/L(t_w)$, for $L_g=400$, 1600, and $z=0.1$, 1, 4, with $z=L(t_w)/L_g$. Solid lines are the exact results for $z\to 0$ and $z\to\infty$.

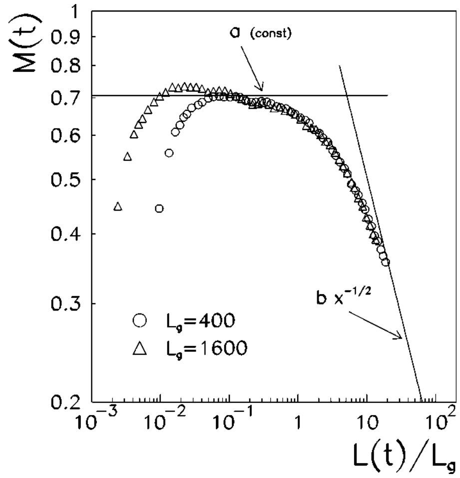

FIG. 4. Total staggered magnetization $M(t)$ versus rescaled average length of domains $L(t)/L_g$, for $L_g=400$,1600.

follows $x=\sqrt{t/t_w}$ and from the exact solution [15] of the Glauber dynamics for the Ising chain

$$F(x,z=0)=\frac{2}{\pi}\arcsin\left(\frac{2}{1+x^2}\right). \tag{23}$$

The second case is obtained taking t_w so large that $\ln L_g$ can be neglected with respect to $\ln t_w$. This yields $x=(\ln t/\ln t_w)^2$ and [13]

$$F(x,z=\infty)=\frac{4}{3\sqrt{x}}-\frac{1}{3x}. \tag{24}$$

In the intermediate cases with finite values of z, one expects

$$F(x,z)=\begin{cases} F(x,0) & \text{for } x-1\ll 1/z \\ F(x,\infty) & \text{for } x-1\gg 1/z \end{cases} \tag{25}$$

where the condition on x can be rewritten more transparently as $L(t)-L(t_w)\ll L_g$ or $L(t)-L(t_w)\gg L_g$. While our data in Fig. 3 show clearly that Eq. (25) is obeyed for $x-1\ll 1/z$, the check of the crossover to the Sinai regime requires simulation times too long for what we can achieve. In any case, the behavior of the data under variation of z in Fig. 3 shows clearly the onset of the crossover. Notice that Eq. (25) means that even if the shortest time t_w is chosen inside the Sinai regime, namely, after the potential has developed a rough landscape, for displacements up to L_g in the bottom of the potential valleys interface diffusion takes place as in the pure system.

Finally, let us consider the behavior of the staggered magnetization

$$M(t,L_g)=\frac{T}{Nh_0^2}E_h\left[\sum_i \langle\sigma_i(t)\rangle_h h_i\right]. \tag{26}$$

Essentially, this quantity gives the behavior of (minus) the magnetic energy per spin. At $t=0$ the spin and field configurations are uncorrelated and $M(t=0,L_g)=0$. As time evolves, one expects the spins to correlate with the field producing growth in $M(t,L_g)$. Indeed, $M(t,L_g)$ displays (Fig. 4) growth in the preasymptotic regime $L(t)\ll L_g$, which however, is followed by an intermediate regime with a constant plateau for $L(t)\sim L_g$ and then by a decrease toward zero for $L(t)\gg L_g$. In the intermediate and asymptotic regimes one has

$$M(t,L_g)=\mathcal{M}\left(\frac{L(t)}{L_g}\right) \tag{27}$$

with $\mathcal{M}(x)\sim x^{-1/2}$ for $x\gg 1$. In order to understand how this comes about, it is useful to look at $M(t,L_g)$ as made up by separate contributions associated to the single interfaces whose number decreases as time goes on. This is formalized by writing

$$M(t,L_g)=\rho_I(t)M_{\text{eff}}(t,L_g), \tag{28}$$

which defines $M_{\text{eff}}(t,L_g)$ as the effective staggered magnetization associated to a single interface. Using Eq. (27) then we have

$$M_{\text{eff}}(t,L_g)=L_g\mathcal{M}_{\text{eff}}\left(\frac{L(t)}{L_g}\right) \tag{29}$$

with (Fig. 5) the function $\mathcal{M}_{\text{eff}}(x)$ obeying

$$\mathcal{M}_{\text{eff}}(x)\sim\begin{cases} x & \text{for } x\ll 1 \\ \sqrt{x} & \text{for } x\gg 1. \end{cases} \tag{30}$$

The interpretation of $M_{\text{eff}}(t,L_g)$ as the single interface contribution to the buildup of magnetization can be substantiated by measuring $M(t,L_g)$ in the process with the initial condition (15). In this case, defining $M_{\text{single}}(t,L_g)=NM(t,L_g)$, we find (Fig. 6)

$$M_{\text{single}}(t,L_g)=L_g\mathcal{M}_{\text{single}}\left(\frac{L(t)}{L_g}\right), \tag{31}$$

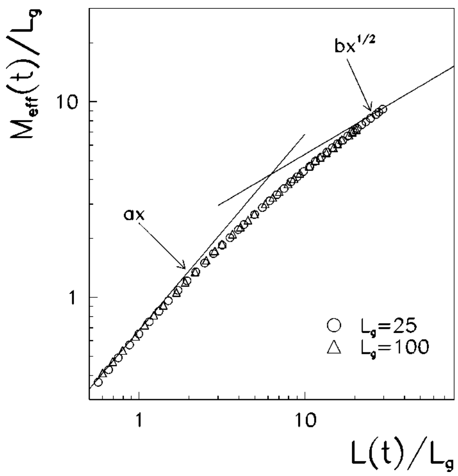

FIG. 5. Rescaled effective staggered magnetization $M_{\mathrm{eff}}(t)/L_g$ versus rescaled average length of domains $L(t)/L_g$, for $L_g=25{,}100$. The fit parameters are $a=0.68$ and $b=1.7$.

where $\mathcal{M}_{\mathrm{single}}(x)$ displays the behavior (30). From this we may draw the following conclusions.

(i) When there is only one interface in the system, the spin-field correlation grows with time. Therefore, this is not a local effect involving only the alignment of the pair of spins at the interface with the local field. Rather, it is a large scale effect involving the optimization of the interface position with respect to the entire field configuration.

(ii) The magnetization growth takes place with different time laws in the preasymptotic and asymptotic regimes.

(iii) The total magnetization behavior of Fig. 4, when multiple interfaces are present, then is just due to the fact that in the preasymptotic regime the rate of growth of the single interface magnetization balances the rate of loss of interfaces, while in the asymptotic regime interfaces do disappear faster than the growth of the single interface contribution to the magnetization.

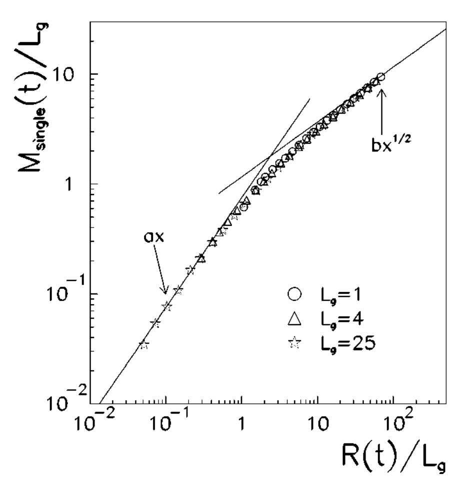

FIG. 6. Rescaled single interface staggered magnetization $M_{\mathrm{single}}(t)/L_g$ versus rescaled RMSD $R(t)/L_g$, for $L_g=1{,}4{,}25$. The fit parameters are $a=0.75$ and $b=1.15$.

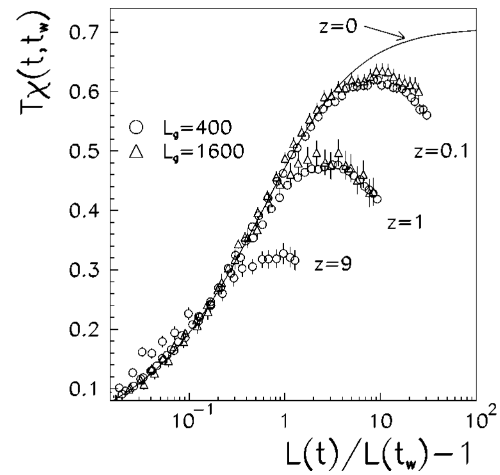

FIG. 7. Total linear response $T\chi(t,t_w)$ versus the ratio $[L(t)-L(t_w)]/L(t_w)$, for $L_g=400$, 1600, and $z=0.1$, 1, 9. The solid line is the exact result for $z\to 0$.

III. RESPONSE FUNCTION

Let us now consider what happens if at the time $t_w>0$ an additional random field $\epsilon_i=\pm\epsilon_0$ uncorrelated with h_i and with expectations

$$E_\epsilon(\epsilon_i)=0$$

$$E_\epsilon(\epsilon_i\epsilon_j)=\epsilon_0^2\delta_{ij} \tag{32}$$

is switched on. We take $\epsilon_0 \ll h_0$ and we are interested in the linear response function with respect to the ϵ perturbation given by

$$T\chi(t,t_w,L_g)=\frac{T}{N\epsilon_0^2}E_hE_\epsilon\left[\sum_i \langle\sigma_i(t)\rangle_{h,\epsilon}\epsilon_i\right], \tag{33}$$

where the external field h acts from $t=0$ and undergoes the change from h to $h+\epsilon$ at t_w.

Due to the suppression of thermal fluctuations within domains enforced through $J=\infty$, the stationary contribution $\chi_{\mathrm{st}}(t-t_w)$ in Eq. (2) vanishes identically. Therefore, the above response function is entirely constituted by the aging term $\chi_{\mathrm{ag}}(t,t_w)$. The interest, then, is focused on the scaling properties. As in the case of the autocorrelation function, we find (Fig. 7) that this quantity obeys the scaling form

$$T\chi(t,t_w,L_g)=\tilde{\chi}\left(\frac{L(t)}{L(t_w)},z\right). \tag{34}$$

For $z=0$ the pure system response function [11] is recovered

$$\tilde{\chi}(x,z=0)=\frac{\sqrt{2}}{\pi}\arctan\sqrt{x^2-1}, \tag{35}$$

which is responsible for the violation of Eq. (4) in the pure Ising chain [12]. In fact, from the equilibrium state (17) we ought to have $P_{\mathrm{eq}}(q)=\delta(q-1)$, which is consistent with Eq. (4), as explained in the Introduction, if only $\hat{\chi}_{\mathrm{st}}(G)$ given

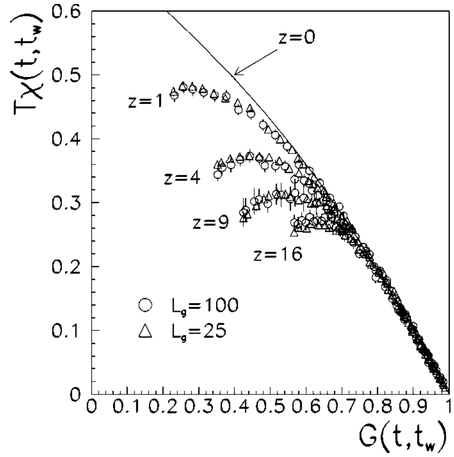

FIG. 8. Total linear response $T\chi(t,t_w)$ versus autocorrelation $G(t,t_w)$, for $L_g=25{,}100$, and $z=1{,}4{,}9{,}16$. The solid line is the exact result for $z\to 0$.

by Eq. (6) enters into $\hat{\chi}(G)$ and the limit $M^2\to 1$ is taken after differentiation. Instead, eliminating x between Eqs. (35) and (23) one finds

$$\hat{\chi}(G,z=0)=\frac{\sqrt{2}}{\pi}\arctan\left[\sqrt{2}\cot\left(\frac{\pi}{2}G\right)\right], \tag{36}$$

which spoils Eq. (4).

With $z>0$ there is a crossover. For values of x up to $x-1\sim 1/z$ Fig. 7 shows that $\tilde{\chi}(x,z)$ behaves as in the pure case, on the basis of the same argument used for the autocorrelation function. For larger values of x the data show that $\tilde{\chi}(x,z)$ levels off and then decreases. This is clearly displayed also in the plot (Fig. 8) against the autocorrelation function. The mechanism responsible for this behavior is the same as discussed in the preceding section for the staggered magnetization. Let us look at the effective response of a single interface $\chi_{\mathrm{eff}}(t,t_w)$ defined in Eq. (8). We find

$$T\chi_{\mathrm{eff}}(t,t_w,L_g)=L(t_w)\tilde{\chi}_{\mathrm{eff}}\left(\frac{L(t)}{L(t_w)},z\right) \tag{37}$$

with the scaling function displaying (Fig. 9) the behavior

$$\tilde{\chi}_{\mathrm{eff}}(x,z)\sim\begin{cases}\tilde{\chi}_{\mathrm{eff}}(x,z=0) & \text{for } x-1\ll 1/z\\ \sqrt{x} & \text{for } x-1\gg 1/z,\end{cases} \tag{38}$$

where $\tilde{\chi}_{\mathrm{eff}}(x,z=0)=x\tilde{\chi}(x,z=0)$. From this follows Eq. (11) in the preasymptotic regime and Eq. (12) in the asymptotic regime, which account for the crossover of the response function in Fig. 7 in terms of the balance between the rate of growth of the single interface response and the rate of loss of interfaces. Hence, for $z>0$ eventually $\tilde{\chi}(x,z)$ vanishes and in the limit $z\to\infty$ one expects

$$\tilde{\chi}(x,z=\infty)\equiv 0. \tag{39}$$

Therefore, for any finite h_0 the validity of Eq. (4) is restored asymptotically.

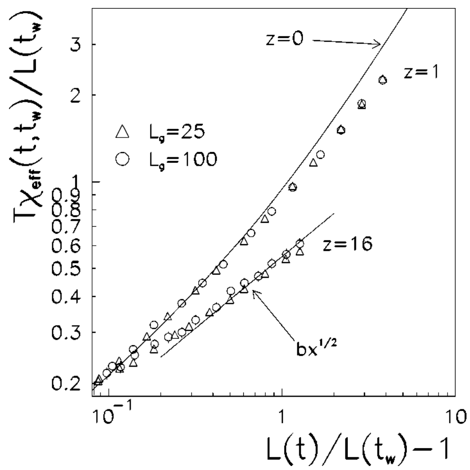

FIG. 9. Rescaled effective linear response $T\chi_{\mathrm{eff}}(t,t_w)/L(t_w)$ versus the ratio $[L(t)-L(t_w)]/L(t_w)$, for $L_g=25{,}100$, and $z=1{,}16$. The solid line is the exact result for $z\to 0$.

Lastly, it remains to make sure that $\tilde{\chi}_{\mathrm{eff}}(x,z)$ may be identified with the response function of a single interface. This we have done by measuring the response function $\chi_{\mathrm{single}}(t,t_w,L_g)$ in the process with only one interface in the initial condition and finding (Fig. 10) a behavior quite close to that of Fig. 9.

IV. CONCLUSIONS

One of the hypothesis for the validity of Eq. (4) relating static and dynamic properties is that the large time limit of $\chi(t,t_w)$ reaches the equilibrium value χ_{eq}. It is then clear from Eq. (2) that for this to be true in the phase-ordering process $\chi_{\mathrm{ag}}(t,t_w)$ must vanish, since this is an intrinsically out-of-equilibrium contribution. In other words, the existence of the interfacial degrees of freedom, which do not equilibrate, must not play a role at the level of the response

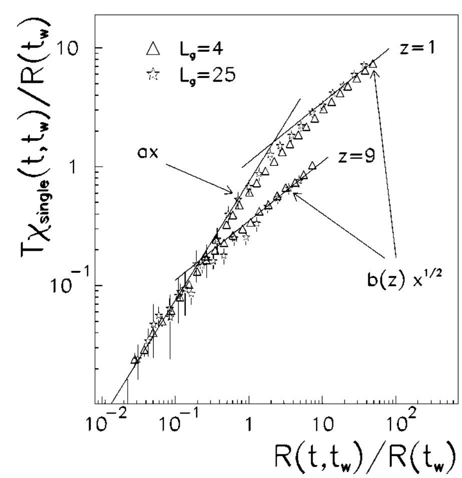

FIG. 10. Rescaled single interface linear response $T\chi_{\mathrm{single}}(t,t_w)/R(t_w)$ versus the ratio $R(t,t_w)/R(t_w)$, where $R(t,t_w)$ and $R(t_w)$ are, respectively, the RMSDs between times t_w and t, and between times 0 and t_w, for $L_g=4{,}25$, and $z=1{,}9$. The fit parameters are $a=0.75$, $b(1)=1.1$, $b(9)=0.35$.

function. Indeed, $\chi_{\mathrm{ag}}(t,t_w)$ normally does vanish. However, there are exceptions in special cases and the study of these is quite instructive since it allows to gain insight into the properties of $\chi_{\mathrm{ag}}(t,t_w)$. One-dimensional systems with a scalar order parameter and frozen bulk thermal fluctuations do make such a special case. In the $d=1$ pure Ising model, due to the pointlike structure of domain walls, the minimization of magnetic energy introduces a bias in the diffusion of interfaces, which leads to a nonvanishing $\chi_{\mathrm{ag}}(t,t_w)$ through Eq. (11). Elsewhere [12] we have analyzed how Eq. (11) ceases to hold in going from $d=1$ to $d>1$, due to the extended nature of interfaces. In that case the minimization of magnetic energy is hindered by the competing need to minimize the curvature of interfaces and $\chi_{\mathrm{ag}}(t,t_w)$ asymptotically disappears. In this paper we have investigated a different mechanism altering the delicate balance (11) between the gain and loss of contributions to the response function, which does not require the passage from pointlike to extended defects. In the $d=1$ RFIM it is the gradual roughening of the landscape that slows down the minimization of the magnetic energy with respect to the growth law of the domain size and which eventually yields an asymptotically vanishing $\chi_{\mathrm{ag}}(t,t_w)$. Therefore, the overall picture that comes out is that a coarsening system is always out of equilibrium in the sense that there are always interfaces around, each one producing the out-of-equilibrium contribution $\chi_{\mathrm{eff}}(t,t_w)$ to the response function, independently of dimensionality or the presence of quenched disorder. These are elements that affect the finer details, such as whether $\chi_{\mathrm{eff}}(t,t_w)$ is a constant or how fast it grows. Where these properties come into play, is in putting together the contributions of all the separate interfaces through $\chi_{\mathrm{ag}}(t,t_w)=\rho_I(t)\chi_{\mathrm{eff}}(t,t_w)$ which, now depending on dimensionality or quenched disorder, may then produce either a vanishing or an asymptotically persistent net result.

Finally, a comment should be made about the noncommutativity of the order of the limits $h_0\rightarrow 0$ and $t_w\rightarrow\infty$. If the limit $h_0\rightarrow 0$ is taken first, the linear response function of the pure Ising model is obtained and, as discussed above, Eq. (4) is violated. Instead, if $t_w\rightarrow\infty$ is taken first one finds the asymptotic linear response function (39) of the RFIM, which is consistent with Eq. (4). However, if the limit $h_0\rightarrow 0$ is taken next, the response function sticks to $\widetilde{\chi}(x,z=\infty)$, which is not the linear response function of the pure model.

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